Year 9 Mathematics Workbook

Level 4

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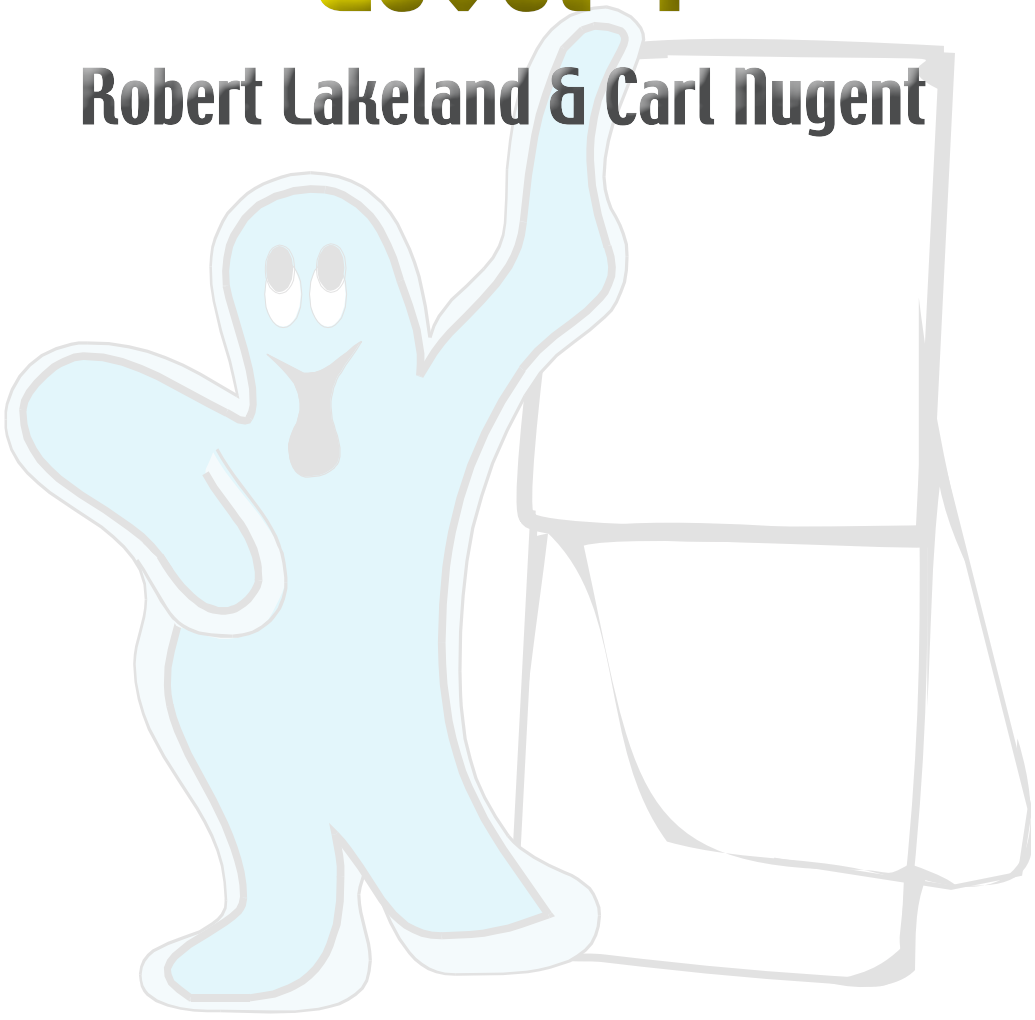

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1.0 Number

1.1 Whole Numbers – Addition and Subtraction

Addition of Whole Numbers using Written Methods

To add whole numbers using written methods we begin by ensuring that the columns line up correctly (i.e. the ones with the ones, the tens with the tens, the hundreds with the hundreds and the thousands with the thousands etc).

For the problem 1024 + 235 + 6 + 198 we set it out as follows:

$$\begin{array}{r} 1024 \\ 235 \\ 6 \\ +\ 198 \\ \hline \end{array}$$

We then begin by adding the right most column. If the column totals more than 9, then we 'carry' the number of tens into the second column. In this case the column total is 23 so we put the 3 in the ones column and carry the 2 (20) to the next column. We continue in this manner right to left from column to column.

$$\begin{array}{r} 1024 \\ 235 \\ 6 \\ +\ 198 \\ \hline 1463 \\ {\scriptstyle 1\ 2} \end{array}$$

Addition of Whole Numbers using a Calculator

Using a calculator we enter the problem from left to right just as it is written down.

For the problem 1024 + 235 + 6 + 198 we enter:

1	0	2	4	+	2
3	5	+	6	+	1
9	8	=			

which gives 1463.

When using a calculator always check that the answer makes 'sense'. Use your estimation skills or number strategies to do the problem in your head. For example, using estimation
1000 + 200 + 10 + 200 = 1410 so we know that 1463 is a realistic answer to the question.

Subtraction of Whole Numbers using Written Methods

To subtract whole numbers using written methods we begin by ensuring that the columns line up correctly (i.e. the ones with the ones, the tens with the tens, the hundreds with the hundreds and the thousands with the thousands etc).

For the problem 4351 – 1492 we set it out as follows:

$$\begin{array}{r} 4351 \\ -\ 1492 \\ \hline \end{array}$$

We begin by subtracting from the right most column, borrowing from the column to the left if necessary. In the first column we cannot subtract 2 from 1 so we borrow 10 from the tens column and subtract 2 from 11. We then change the 5 in the tens column (50) to a 4 (40). We continue in this manner right to left from column to column.

$$\begin{array}{r} {\scriptstyle 3\ 12\ 14\ 11} \\ \not{4}\ \not{3}\ \not{5}\ \not{1} \\ -\ 1\ 4\ 9\ 2 \\ \hline 2\ 8\ 5\ 9 \end{array}$$

Subtraction of Whole Numbers using a Calculator

Using a calculator we enter the problem from left to right just as it is written down.

For the problem 4351 – 1492 we enter:

4	3	5	1	–	1
4	9	2	enter		

which gives 2859.

When using a calculator always check that the answer makes 'sense'.

Use your estimation skills or number strategies to do the problem in your head.

For example, using estimation
4400 – 1500 = 2900 so we know that 2859 is a realistic answer to the problem.

When using a calculator it is a good idea to do the problem twice, ensuring you get the same answer both times. This reduces the likelihood of 'key in' error.

Achievement – Evaluate the following using your calculator. Do each problem twice, as a check, to reduce the likelihood of 'key in' error.

31. 2754 x 31 =

32. 45 812 x 15 =

33. 5671 x 142 =

34. 2435 x 124 =

35. 23 x 14 x 25 =

36. 145 x 27 x 38 =

37. 57 x 104 x 319 =

38. 249 x 316 x 478 =

39. 67 993 x 2 451 =

40. 5784 x 204 =

41. 3768 ÷ 3 =

42. 14 166 ÷ 9 =

43. 16 325 ÷ 5 =

44. 7470 ÷ 6 =

45. 1612 ÷ 13 =

46. 5015 ÷ 17 =

47. 39 324 ÷ 29 =

48. 193 572 ÷ 76 =

49. 203 040 ÷ 54 ÷ 16 =

50. 1 084 005 ÷ 65 ÷ 17 =

Achievement – Evaluate the following using your calculator. Do each problem twice, as a check, to reduce the likelihood of 'key in' error.

97. 34.5 x 1.4 =

98. 26.72 x 2.6 =

99. 123.1 x 0.2 =

100. 436.8 x 6.8 =

101. 14.58 x 3.4 =

102. 2.891 x 0.56 =

103. 2.45 x 1.5 x 3.55 =

104. 204.95 x 0.6 =

105. 145.95 x 0.6 x 1.2 =

106. 982.3 x 14.1 =

107. 12.6 x 4.8 x 8.6 =

108. 3.65 x 14.6 x 28.2 =

109. 567.85 ÷ 0.5 =

110. 863.6 ÷ 0.04 =

111. 1443.465 ÷ 1.5 =

112. 6196.48 ÷ 0.94 =

113. 96 ÷ 3.75 =

114. 283.464 ÷ 2.48 =

115. 4603.24 ÷ 15.7 =

116. 2637.36 ÷ 1.85 =

117. 897.875 ÷ 0.275 =

118. 9544.19 ÷ 1.45 =

Merit – Answer the following application problems, rounding to the required accuracy.

167. Taylor works 45 hours per week and earns $25 per hour. How much does he earn to the nearest ten dollars each week?

168. Sam has a loan of $10 250 from the bank. He pays $645 per month. How long to the nearest month will it take him to pay his loan off?

169. Drew travels 485 km per day as a courier. He works 5 days a week for 49 weeks of the year. How far does he travel to the nearest 10 000 km per year?

170. A company's quarterly profit is listed as $8 455 000. How much profit will they make for the financial year to the nearest 1 000 000 if their profit per quarter stays the same?

171. An airline's income from a single flight to London is $443 325 less expenses of $275 350. What profit to the nearest $10 000 does the airline make?

172. A company sells 35 250 books at $25 each in one year. What is their gross income to the nearest 10 000?

173. The population of Australia currently (2009) is estimated to be 21 613 115. What is the population of Australia to the nearest million?

174. A syndicate of six people win Lotto first division valued at $1 250 480. How much would each person in the syndicate get rounded to the nearest $10 000?

175. A female lives on average for 43 256 880 minutes. How long does an average female live to the nearest day?

176. A relative leaves a $5 million dollar estate to be divided evenly among an extended family of 24. How much does each person get to the nearest $1000?

177. If every 2 Canadian dollars is equal to 3 NZ dollars, how many NZ dollars to the nearest 100 would you get for 16 500 Canadian dollars?

178. A person wins a lottery which pays out $235 000 per year for 25 years. How much does the person receive in total to the nearest 1 000 000?

179. If every US dollar is equal to 2 NZ dollars, how many US dollars to the nearest 10 would you get for 3425 NZ dollars?

180. A company makes $4 350 285 profit over a 12 month period. How much per day does this equate to, to the nearest 1000?

181. A population of insects doubles everyday. If the population size is initially 20, what will be the population to the nearest 100 insects in 5 days?

182. A marathon runner can run one kilometre in 4 minutes. How long to the nearest ten minutes would it take him to complete a marathon (42 km)?

Use +, –, x, ÷, brackets, exponents as well as square roots with the digits 1, 9, 4 and 5 to make up questions to give the answers from 0 to 50. You can use each of the digits 1, 9, 4, and 5 in any order, but only once in each question. The first one is done for you. Put each of your answers in the appropriate cell of the table drawn below. Note. There is more than one correct answer for each number.

$9^1 - 4 - 5 = 0$	= 17	= 34
= 1	= 18	= 35
= 2	= 19	= 36
= 3	= 20	= 37
= 4	= 21	= 38
= 5	= 22	= 39
= 6	= 23	= 40
= 7	= 24	= 41
= 8	= 25	= 42
= 9	= 26	= 43
= 10	= 27	= 44
= 11	= 28	= 45
= 12	= 29	= 46
= 13	= 30	= 47
= 14	= 31	= 48
= 15	= 32	= 49
= 16	= 33	= 50

Achievement – Evaluate the following.

325. $\frac{2}{3}$ of 21

326. $\frac{6}{7}$ of 14

327. $\frac{2}{9}$ of 72

328. $\frac{5}{8}$ of 32

329. $\frac{7}{10}$ of 100

330. $\frac{5}{6}$ of 96

331. $\frac{\square}{7}$ of 35 = 10

332. $\frac{2}{\square}$ of 40 = 16

333. $\frac{3}{8}$ of $\square$ = 15

334. $\frac{\square}{5}$ of 60 = 48

335. $\frac{4}{\square}$ of 36 = 16

336. $\frac{5}{11}$ of $\square$ = 30

Merit – Evaluate the following, showing a line of working to indicate the calculation you are doing.

337. When a box of eggs is dropped two-thirds of them are broken. If the box holds 18 eggs, how many are broken?

338. One hundred and fifty people take part in a survey. Three-fifths of them are female. How many females are there?

339. Taylor spends ninety-five minutes on his homework. Three-fifths of this time he spent on maths. How long did Taylor spend on maths?

340. A box contains forty-eight glasses, one sixth of which have a manufacturing flaw. How many glasses are flawed?

341. The ingredients for an apple crumble for four people is given in the table below. Complete the table to find the quantity of ingredients for the same recipe for three and five people.

Ingredients for four people	Ingredients for three people	Ingredients for five people
320 g of apple		
56 g of margarine		
112 g of flour		
56 g of sugar		
20 g of fruit juice		
4 g of cinnamon		

Merit – Answer the following integer multiplication and division application problems.

512. If Simon spends \$165 more than he earns each month and his current bank balance is \$850, what will be his balance in 6 months time?

513. Four brothers decide to purchase a bach costing \$245 000. They have \$62 000 for a deposit and have to borrow the rest? What is the balance of each of the brothers' loan account after purchasing the bach?

514. A debt of \$230 000 ($^-$\$230 000) is to be shared evenly among 8 people. How much of the debt is each person responsible for?

515. A business is losing \$440 per month. If their current bank balance is \$1500, what will be their balance in 12 months time?

516. An oil rig has to drill to a depth of 384 m ($^-$384 m). If it can drill at a rate of 32 m per day, how many days before it will reach the required depth?

517. A temperature probe registers a temperature of $^-$124° C. If the probe's temperature increases by 8 °C per hour, after how many hours will it be at a temperature of 4° C?

518. A company's debt accumulates at a rate of \$1.25 million per month ($^-$\$1.25 m).

a) What would be the company's annual debt?

b) If the annual debt was to be shared evenly between eight directors, what is each director's liability?

519. A company's current account after one year in business stands at $^-$\$3.45 million. If their average profit per month for the next twelve months is \$450 000, what would be the company's new current account balance at the end of the next financial year?

520. The Mariana Trench is 10 911 metres below sea level ($^-$10 911 m), three times more than the depth of the Hikurangi Trench. What is the depth of the Hikurangi Trench?

521. Laguna del Caban in Argentina is 105 metres below sea level ($^-$105 m). The Dead Sea Shore in Israel is four times lower. What is its depth?

522. The balance of Jason's overdraft is \$2450 OD ($^-$\$2450). If Jason's repayments are \$105 per month what will be the balance of his overdraft after 18 months?

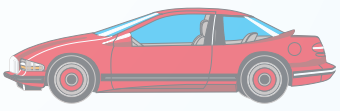
523. Kay has a student loan of \$8950 ($^-$\$8950). If she borrows a further \$9750 per year for three years what will be the balance of her loan when she has completed her course?

Find the answer to each of the following percentage questions. Join the question and answer with a straight line. Each line will pass through a letter. Enter the letter in the appropriate spot at the bottom of the page to answer the riddle.

Write 48% as a simplified fraction	26.7% (1 dp)
Write 0.025 as a percentage.	35%
Convert 15.8% to a decimal.	37.5%
Express 12 out of 45 as a percentage.	$\frac{12}{25}$
Jason earned \$80 and paid \$16 tax. What percentage of his earnings did Jason pay in tax?	42
Find 35% of 120.	\$15.60
What percentage of 48 is 15?	2.5%
Sally earns \$390 a week but is about to get a 4% pay rise. How much does she now get extra a week?	31.25%
What percentage of 90 is 31.5?	47% (0 dp)
12 out of 32 students in a class bike to school. What is this as a percentage?	0.158
Convert $\frac{7}{15}$ to a percentage.	20%

O C I T N A N I A R N

What would a country be called if everyone in it lived in their cars?

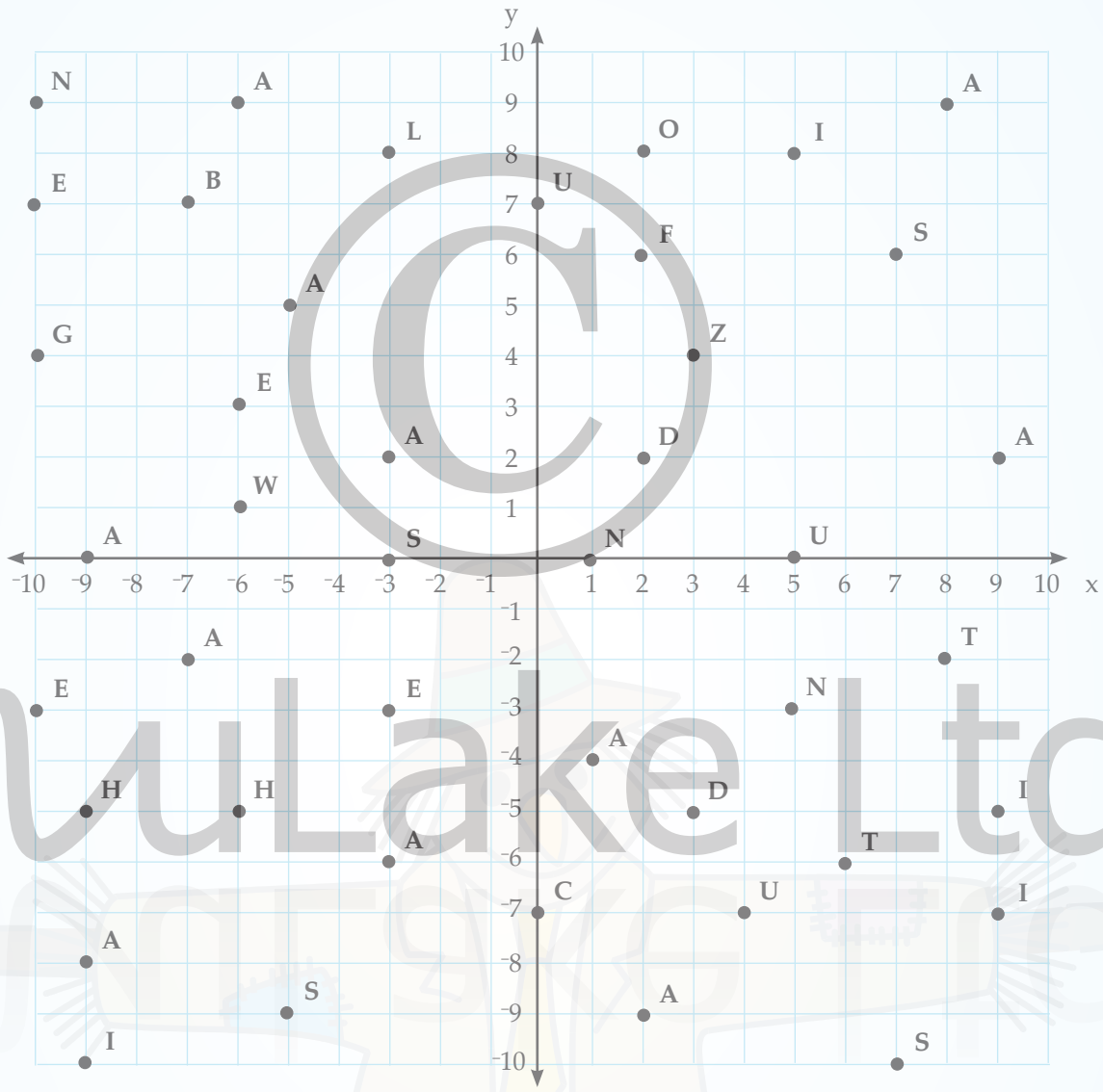


31.25% 35%

— —

2.5%	42	$\frac{12}{25}$	37.5%	20%		47%	37.5%	0.158	15.6	26.7%	47%

Find the answer to the riddle at the bottom of the page by identifying which letter corresponds to the applicable coordinate. For example the coordinate (3, 4) corresponds to the letter Z.



Why did the scarecrow win the Nobel Prize?

(-7, 7)	(-6, 3)	(0, -7)	(-3, -6)	(5, 0)	(7, 6)	(-10, -3)	(-6, -5)	(-3, -3)	(-6, 1)	(8, 9)	(7, -10)	
(2, 8)	(4, -7)	(8, -2)	(7, -10)	(6, -6)	(-9, -8)	(5, -3)	(2, 2)	(5, 8)	(1, 0)	(-10, 4)		
(9, -5)	(-10, 9)	(-9, -5)	(9, -7)	(-3, 0)	(2, 6)	(-9, -10)	(-10, 7)	(-3, 8)	(3, -5)			

Linear Equations of the form ax = b

Equations of the form ax = b can be solved by dividing both sides of the equation by 'a'.

Consider $5x = 35$

Since 5x means 5 times x, then the equation 5x = 35 means *'five times what number equals 35'*.

Remember an equation is like a set of scales that must always remain in balance. The centre or balance point of the 'equation scales' is the equal sign.

An equation will stay in balance as long as what we do to one side of the equation we also do to the other side.

For the equation 5x = 35 we divide both sides by 5, because we wish to undo the multiply by 5 and therefore just leave x on its own on one side and a number on the other.

So

$$5x = 35$$

$$\frac{5x}{5} = \frac{35}{5} \quad \text{(divide both sides by 5)}$$

$$x = 7$$

Another way of setting out the equation is as below. Both sides have to be divided by 5 and an 'equals' line has been drawn underneath.

$$\begin{array}{r} 5x = 35 \\ \div 5 \quad \div 5 \\ \hline x = 7 \end{array}$$

Dividing down

If the 'a' value in the equation is negative, e.g. $^{-}3$, we must divide both sides by $^{-}3$ NOT just 3.

Example

Solve the equations

a) $3x = 24$

b) $^{-}7x = 63$

a)

$$3x = 24$$

$$\frac{3x}{3} = \frac{24}{3} \quad \text{(divide both sides by 3)}$$

$$x = 8$$

b)

$$\begin{array}{r} ^{-}7x = 63 \\ \div\ ^{-}7 = \div\ ^{-}7 \\ \hline x = ^{-}9 \end{array}$$

Dividing down

Always check your answer, after solving an equation, by substituting it back into the original equation to confirm it is correct.
For the examples above:

a) $3 \times 8 = 24$ ✓

b) $^{-}7 \times ^{-}9 = 63$ ✓

Achievement – Write an expression for each of the following.

134. The sum of 5 and x.

135. Twelve is added to e.

136. Six is taken away from b

137. The sum of a, b and c.

138. k is increased by 8.

139. Five is decreased by x.

140. The difference between n and 4.

141. The difference between 4 and n.

142. Half of x then add 4.

143. One-quarter of p then minus 5.

144. Double n then add 7.

145. Double x then subtract 3.

146. Add 2 to x, then divide by 5.

147. Four times x minus 7.

148. Subtract 4 from twice the number n.

149. Add 6 to the number x and then multiply by 3.

150. Subtract 2 from n and then multiply by 7.

151. Add a, b and c and then divide by 2.

152. The difference of x and y then multiplied by z.

153. The difference of x and y then divided by z.

154. Subtract 15 from three times a number, n.

155. Multiply the number n by itself and add 3.

156. The product of the number x, y and z.

157. Increase n by 3 and then multiply by 5.

Achievement – Evaluate each expression using the values given.

186. a + b; when a = 15 and b = 18.

187. 2x + y; when x = 13, y = 4.

188. 4c; when c = $^{-}$9.

189. x + y + z; when x = 8, y = $^{-}$9 and c = 5.

190. ab; when a = $^{-}$2, b = 4.

191. abc; when a = $^{-}$3, b = $^{-}$5 and c = 4.

192. x + 8 – y; when x = 5 and y = 9.

193. x + 2y + z; when x = 3, y = $^{-}$2 and z = 4

194. 3(a + b); when a = $^{-}$5 and b = 3

195. a – 5c; when a = 7 and b = $^{-}$3

196. 2(a – b – c); when a = 3, b = $^{-}$6 and c = 9.

197. $a^2 + b^2$; when a = 5 and b = $^{-}$9.

198. $\sqrt{x} + y$; when x = 16 and y = $^{-}$8.

199. $\sqrt{x} + y^2$; when x = 25 and y = $^{-}$3.

200. $\frac{a}{3} + 2ab$; when a = 9 and b = 2.

201. abc ÷ 3; when a = 4, b = 9 and c = $^{-}$2.

202. $(a - 2)^2$; when a = $^{-}$6.

203. $3(x + y)^2$; when x = 6 and b = 7.

204. $(x + 3)^2 + b$; when x = 4 and b = 5.

205. $4(x - 5)^2 - y$; when x = 2 and y = $^{-}$2.

206. $2x^2y$; when x = 5 and y = $^{-}$3.

207. 2(x + 4)(x – 5); when x = 3.

Example

Change the following to the unit indicated in brackets.

a) 1.2 ha (m^2)
b) 4300 mm^2 (cm^2)
c) 15 800 m^2 (ha)
d) 25 cm^2 (mm^2)

a) Since there are 10 000 m^2 in a hectare and we are going from a larger to a smaller unit we multiply by 10 000.

1.2 ha x 10 000 = 12 000 m^2

b) Since there are 100 mm^2 in 1 cm^2 and we are going from a smaller to a larger unit we divide by 100.

4300 mm^2 ÷ 100 = 43 cm^2

c) Since 1 hectare is equivalent to 10 000 m^2 and we are going from a smaller to a larger unit we divide by 10 000.

15 800 m^2 ÷ 10 000 = 1.58 ha

d) Since there are 100 mm^2 in 1 cm^2 and we are going from a larger to a smaller unit we multiply by 100.

25 cm^2 x 100 = 2500 mm^2.

Achievement – Change the following to the unit indicated in brackets.

23. 35 000 m^2 (ha) *Smaller to larger, so divide*

24. 9.2 m^2 (cm^2) *Larger to smaller, so multiply*

25. 750 mm^2 (cm^2) *Smaller to larger, so divide*

26. 4.5 cm^2 (mm^2) *Larger to smaller, so multiply*

27. 2.35 ha (m^2)

28. 1200 m^2 (ha)

29. 0.75 m^2 (cm^2)

30. 0.025 m^2 (mm^2)

31. 65 400 000 mm^2 (m^2)

32. 12 800 mm^2 (m^2)

1 ha = 10 000 m^2
1 m^2 = 10 000 cm^2
1 m^2 = 1 000 000 mm^2
1 cm^2 = 100 mm^2

Find the answer to each conversion between units question. Join the question and answer with a straight line. Each line will pass through a letter. Enter the letter in the appropriate spot at the bottom of the page to answer the riddle.

Convert 4.5 m to centimetres	5400
Convert 5.4 km to metres	4500
Convert 0.54 T to kilograms	54 000
Convert 0.045 km to centimetres	450
Convert 4.5 millilitres to cm^3	5.4
Convert 5400 cm to mm	540
Convert 45 500 kg to tonnes	540 000
Convert 0.0054 L to millilitres	54
Convert 0.000 45 cm^2 to mm^2	4.5
Convert 0.054 m^3 into litres	0.45
Convert 45 000 g to kilograms	0.045
Convert 5.4 km to centimetres	45
Convert 4500 m^2 to hectares	45.5

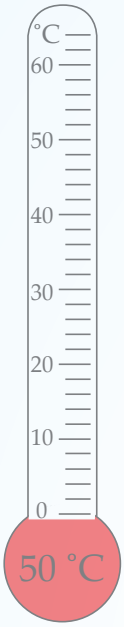
H O N A D Y G U E L R I W

What did the tie say to the hat?

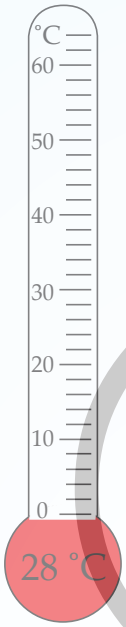
4.5 450 45.5 5.4 450 54 000 5400 54 000 4500 540 54 000 4500 540 45 540 000 540 000 5400 54 000 4500 5.4 54 000 0.045 450 45.5 4500 540 54 000 0.45 5400 45 540 000 54

Achievement – Mark the required temperature on each of the following thermometers, by shading up to the correct position.

106.

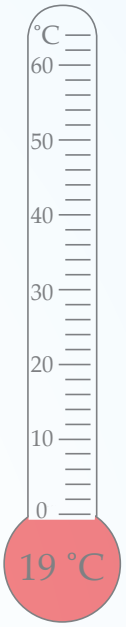


107.



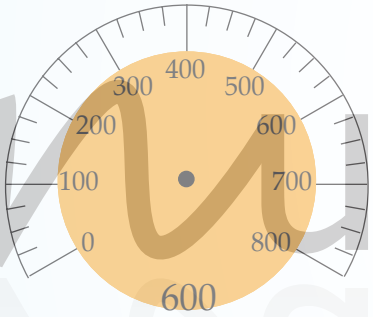
108.

109.



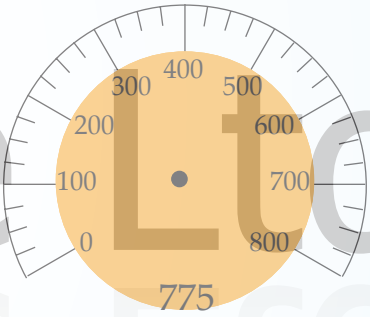
Mark the required value on each of the following gauges, by drawing an arrow from the centre of the gauge to the correct position.

110.



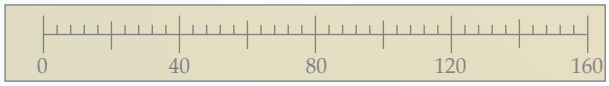
111.

112.

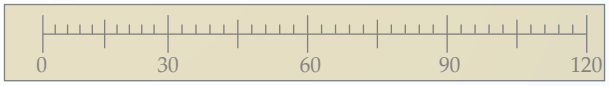


Mark the required value on each of the following rulers, by drawing an arrow pointing to the correct position.

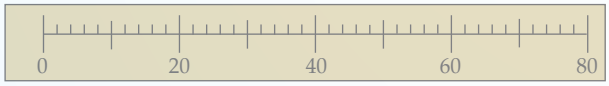
113. Mark the value 100.



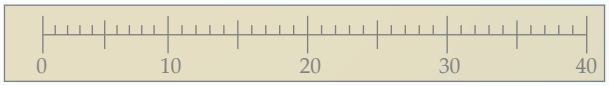
114. Mark the value 36.



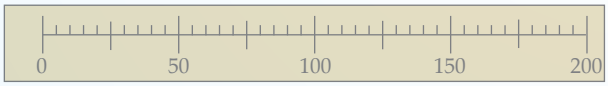
115. Mark the value 57.



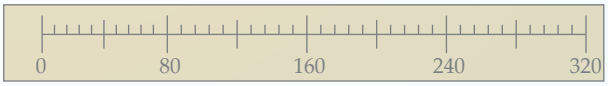
116. Mark the value 21.



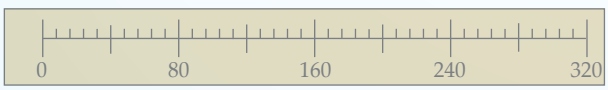
117. Mark the value 95.



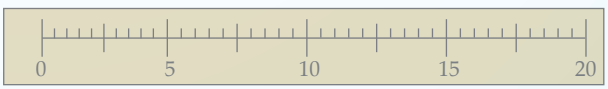
118. Mark the value 172.



119. Mark the value 264.



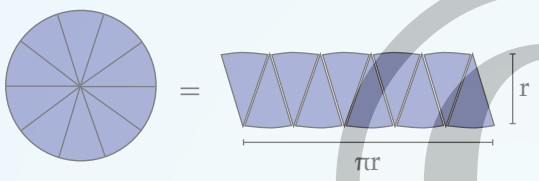
120. Mark the value 13.5.



Area of a Circle

We can use our knowledge of the area of a parallelogram and the circumference of a circle to formulate a rule for the area of a circle.

Consider the circle below which has been divided into 10 sectors. If we then 'unwrap' these sectors from the circle we obtain an approximate parallelogram with height the radius of the original circle and base πr.



Remember that the circumference of a circle is πd or 2πr, therefore half the circumference is πr.

Obviously the top and base of the parallelogram formed are not straight sides, but if we were to divide the circle into 1000 sectors and then unwrap these the top and base would be closer to having a straight top and base.

In summary the formula for the area of a circle is

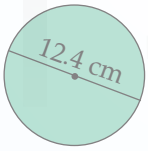
$$\text{Area(circle)} = \pi \times r \times r$$
$$= \pi r^2$$

Example

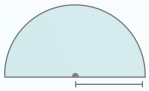
Find the area of the following.

a)

b)



c)



If you are required to find the area of a circle and are given the diameter of the circle, remember to half it to get the radius, before using the formula $A = \pi r^2$.

a) Area (circle) $= \pi r^2$
$= \pi \times 2.50 \times 2.50$
$= 19.6 \text{ m}^2$ (3 sf)

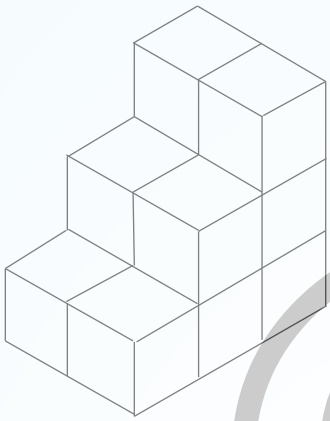
b) Area (circle) $= \pi r^2$
$= \pi \times 6.20 \times 6.20$ (half diameter)
$= 121 \text{ cm}^2$ (3 sf)

c) Area (half-circle) $= \pi r^2 \div 2$
$= (\pi \times 4.3 \times 4.3) \div 2$
$= 29 \text{ mm}^2$ (2 sf)

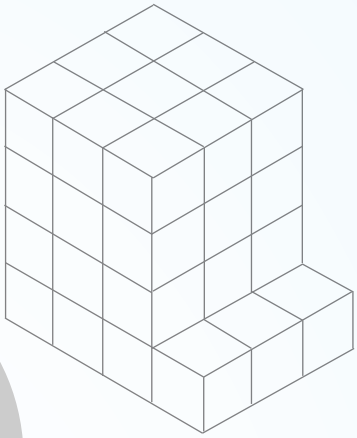
To find the area of a semi-circle (half a circle) we find the area of a circle and divide by 2. See example c) above.

Achievement – Find the volume of the following by counting cubes. Each cube measures 1 cm^3.

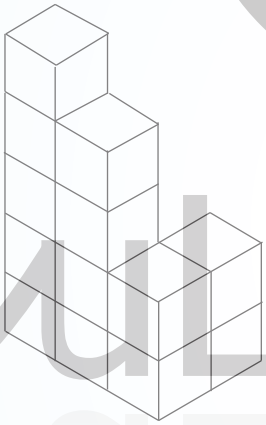
220.



221.

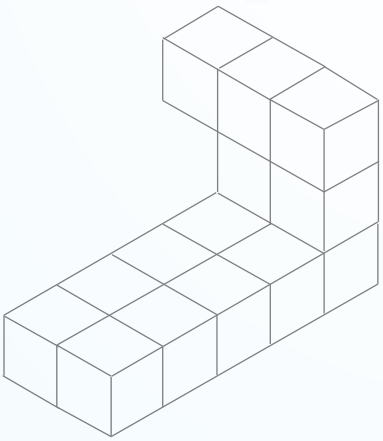


222.



223.

224.



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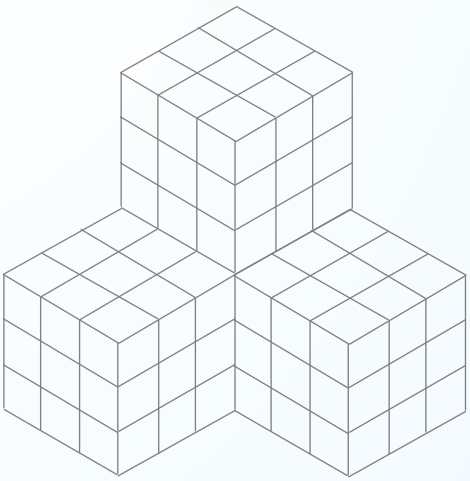


293. Study the TV One timetable on the right and answer the questions relating to it.

a) What time does The Forgotten start. Give your answer in 12-hour time?

b) What is the duration of the programme Criminal Minds?

c) How many times does Te Karere play on Thursday and at what times?

d) Give the total time assigned to TV News on TV One on Thursday?

e) If the program Martina Cole: Girl in Gangs has duration 1 hour 45 minutes when will it end? Give your answer using 24-hour clock time.

f) What is the total time in hours and minutes that Te Karere screens for on a Thursday?

TV One – Thursday	
0000	Outrageous Wasters (G)
0055	Te Karere
0120	BBC World
0535	Te Karere
6000	NZI Business
0630	Breakfast
9000	Good Morning
1200	One News at Midday
1230	Emmerdale (G)
1330	Airline (G)
1400	Dickinsons Real Deal (G)
1500	60 Minute Makeover (G)
1600	Te Karere
1630	One News at 4.30
1700	Ellen (G)
1800	One News at 6pm
1900	Close Up
1930	Coronation Street (PGR)
2030	The Forgotten (AO)
2125	Criminal Minds (AO)
2220	One News Tonight
2250	Tagata Pasifika (G)
2350	Martina Cole: Girl in Gangs (AO)

294. Study the school timetable below right and answer the questions relating to it.

a) What time is morning briefing?

b) What is the length of each period?

c) How long a break is there between period 4 and period 5?

d) How many periods of English are there in a week?

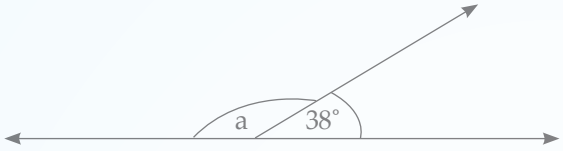
e) What is the length of a school day from the start of briefing to the end of period 7?

f) A teacher at the school has five duties per week. One before school from 0745 to the start of briefing, two half break duties and two half lunch duties. How long does a teacher spend on duty each week?

WEEK A						
Period	Time	Monday	Tuesday	Wednesday	Thursday	Friday
	08:10	*Briefing*	*Briefing*	*Briefing*	*Briefing*	*Briefing*
	08:20-08:25	*Registration*	*Registration*	*Registration*	*Registration*	*Registration*
1	08:30-09:25	History	Art	French	History	French
2	09:30-10:25	Spanish	Science	Geography	Music	English
	10-25-10-45	BREAK				
3	10:50-11:45	PHE	Maths	Science	Maths	Science
4	11:50-12:45	Drama	English	Spanish	ICT	
	12:45-13:40	LUNCH				
	13:40-13:50	REGISTRATION				
5	13:50-14:45	Maths	Spanish	Team/PA	PHE	PSE
6	14:50-15:45	Team/PA	Geography	Team/PA	Science	Activities
7	15:50-16:45	Assembly	Activities	Team/PA	English	Activities

Achievement – Calculate the missing angles.

23.



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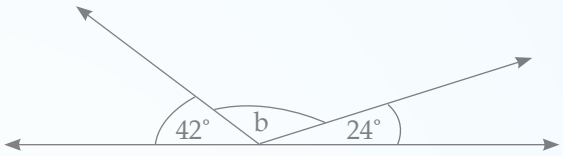

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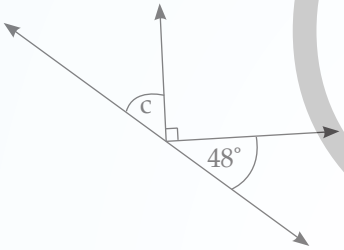

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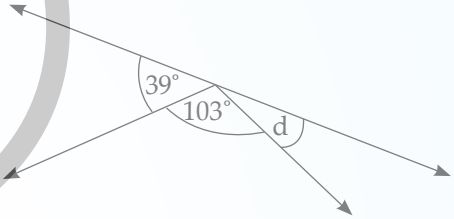

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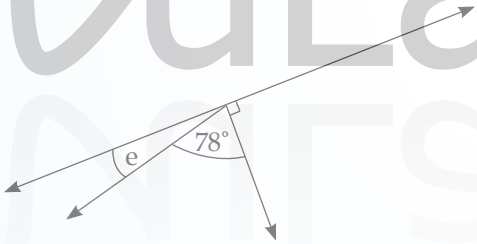

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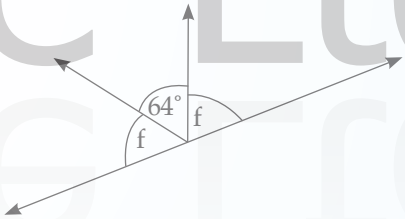

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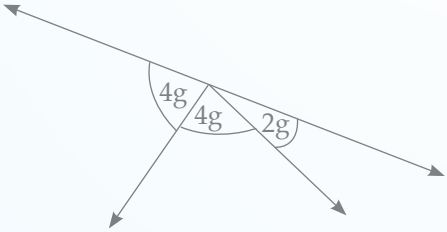

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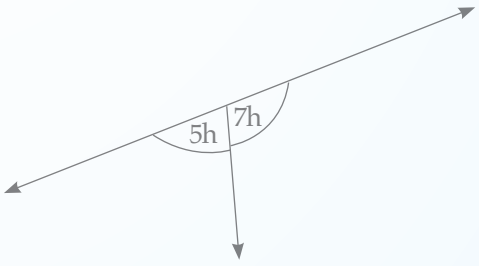
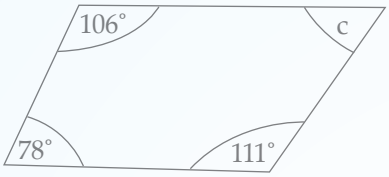


Achievement – Calculate the missing angles.

64.



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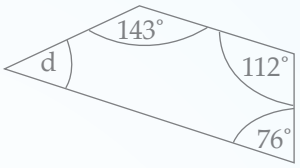

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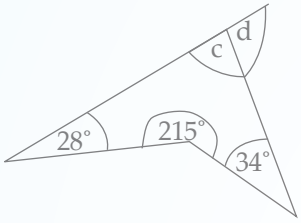

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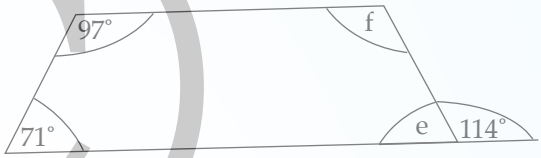

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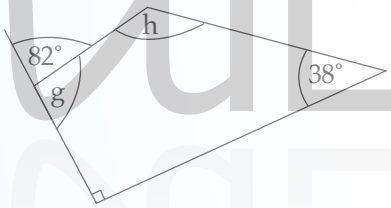

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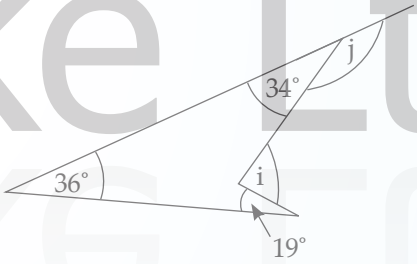

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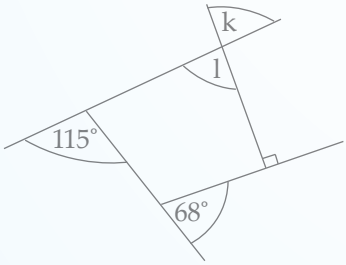

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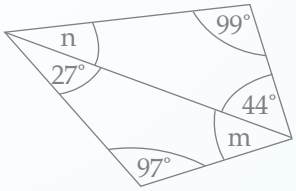
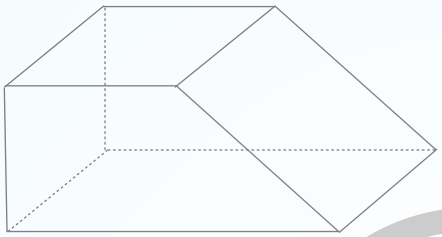


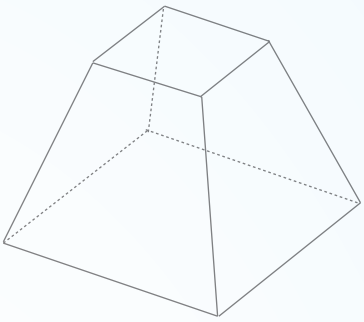
Achievement – Draw nets for the following three-dimensional shapes.

108.



A trapezoidal prism.

109.

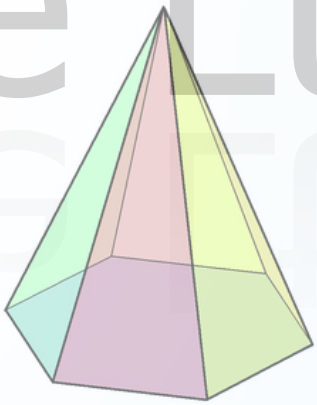


A quadrilateral frustum with a square base.

110.

A triangular prism.

111.



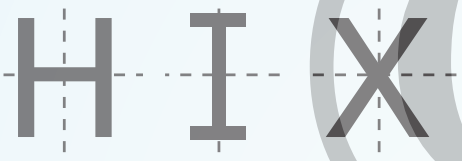
A hexagonal pyramid.

Line Symmetry

An object is symmetrical if a mirror line or lines can be placed on the figure reflecting the object back onto itself.

The three letters of the alphabet below have one axis of symmetry.

These three letters of the alphabet have two axes of symmetry.



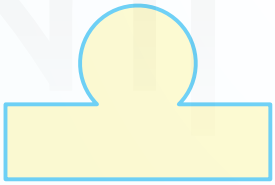
The letter X could have four axes of symmetry, if it was drawn as below.

If we trace a shape onto paper and can then fold it so that one half of the object maps exactly onto the other half, then the fold line is an axis of symmetry.

Example

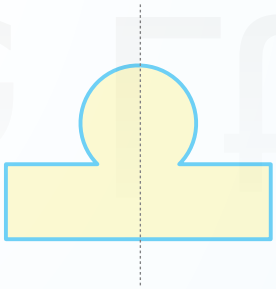
Draw in the axe(s) of symmetry for the following shapes.

a)



a)

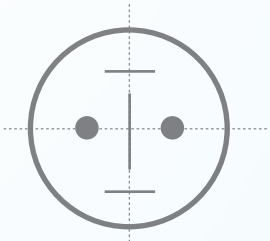
One axis of symmetry.



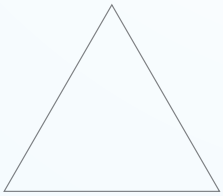
b)

b)

Two axes of symmetry.

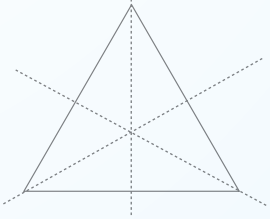


c)



c)

Three axes of symmetry.



Enlargement

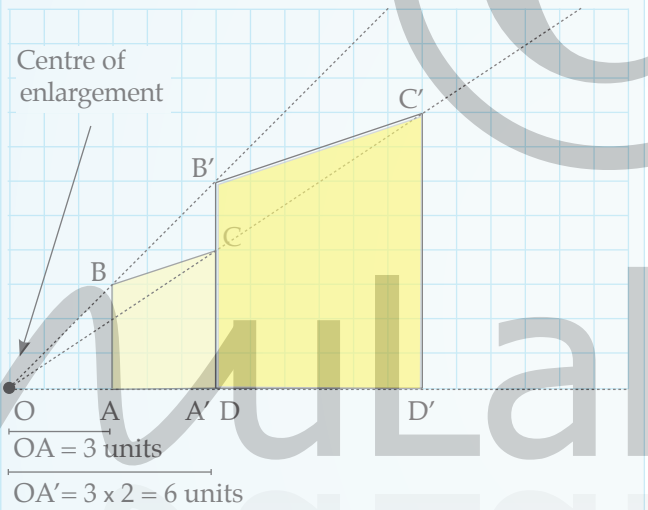
Enlargement is the process of enlarging or reducing a shape or figure about a point called the centre of enlargement, using a given scale factor.

When the scale factor is greater than one the shape is enlarged.

When the scale factor is less than one the shape is reduced.

When we enlarge or reduce an object or shape the length of the lines and the area of the shape change, but the angle sizes and orientation remain unchanged (invariant).

Consider the enlargement below. The figure ABCD has been enlarged by a scale factor of 2. Each length is twice its original size.



An easy way to draw an enlargement is to begin by drawing guide lines from the centre of enlargement through each of the key points of the shape.

The key points of the enlargement when drawn must fit on these guide lines just like the original shape. To calculate the location of the key points of the enlarged figure always count from the centre of enlargement.

For example in the enlargement above the distance from the centre of enlargement to the point A is 3 units. If we multiply this by the scale factor 2, the point A′ must be 6 units (2 x 3) from the centre of enlargement directly along the guide line from the centre of enlargement through the point A.

From the point A′ it is then just a matter of drawing the lengths of the enlarged shape remembering to multiply the originally lengths by 2 to find what the image lengths need to be.

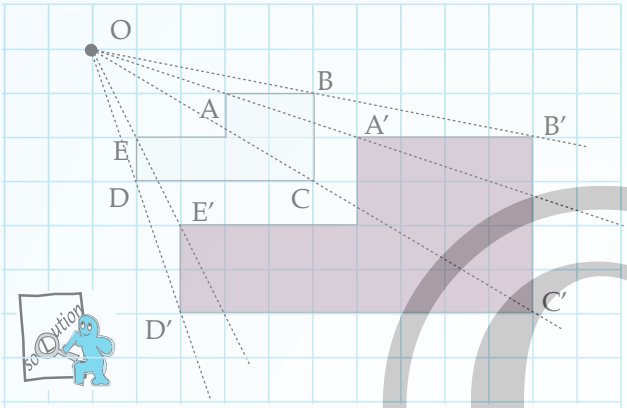
The centre of enlargement remains invariant (unchanged) under enlargement.

If the original figure in a transformation is labelled ABC then we label the transformed image A′B′C′.

You can find the scale factor of an enlargement by inspection or by finding the measurement of an image length and dividing it by the measurement of the original length, e.g. $\frac{\text{length A'D'}}{\text{length AD}} = \frac{6}{3} = 2$**, so the scale factor of the enlargement is 2.**

Achievement – For each of the following enlargements draw in the guide lines through each pair of key points to find the centre of enlargement. Mark it with the letter O. Also find the scale factor of the enlargement. The first one is done for you.

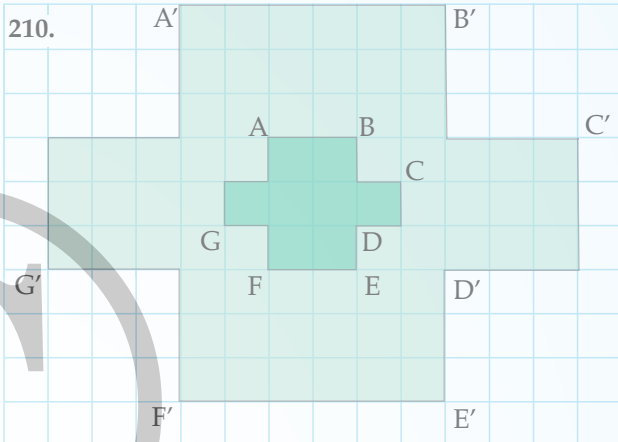
Example



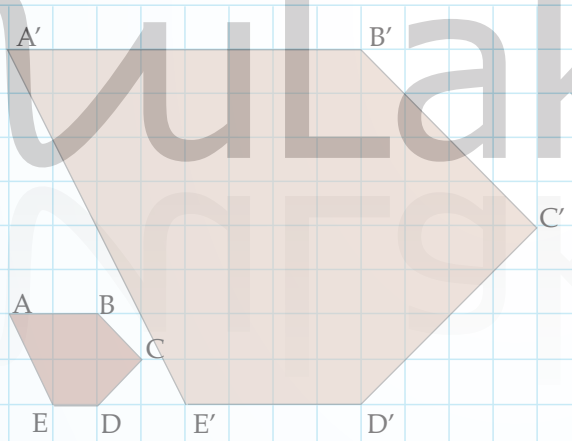
By drawing guide lines through each pair of key points, AA′, BB′ etc. and then finding where they intersect we can locate the centre of enlargement, O.

To find the scale factor of the enlargement we divide an image length by the equivalent original length, i.e. Scale factor $= \frac{\text{length C'D'}}{\text{length CD}} = \frac{8}{4} = 2$.

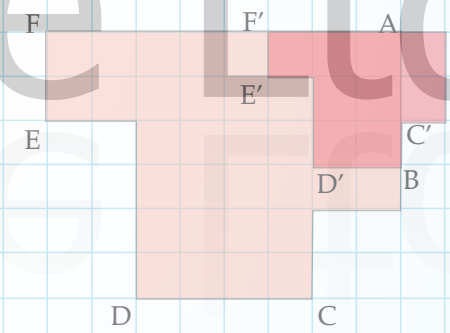
210.



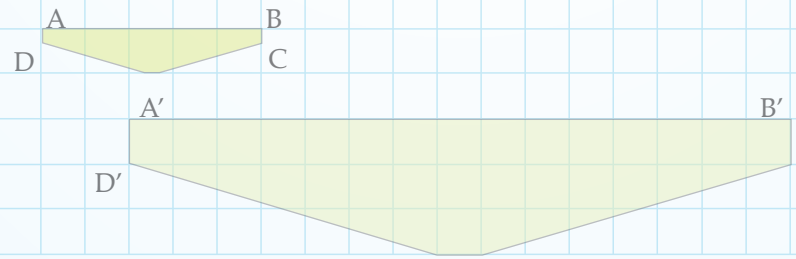
211.



212.



213.



Calculate the mode of:

16. 1, 2, 2, 3, 3, 4, 4, 4, 5, 5, 6, 7, 9, 10, 13, 18, 20, 23, 25, 27

17. 23, 34, 12, 45, 23, 56, 34, 23, 34, 58, 13, 17, 34, 78, 3

18. 3.7, 2.5, 1.7, 5.8, 3.6, 2.0, 9.2, 5.8, 6.4, 17.3, 4.4

Calculate the median of

19. 1, 2, 2, 3, 3, 4, 4, 4, 5, 5, 6, 7, 9, 10, 13, 18, 20, 23, 25, 27

20. 23, 34, 12, 45, 23, 56, 34, 23, 34, 58, 13, 17, 34, 78, 3

21. 3.7, 2.5, 1.7, 5.8, 3.6, 2.0, 9.2, 5.8, 6.4, 17.3, 4.4

Merit – Answer the following questions.

22. Seven people were asked how far they lived from school in kilometres. The results were 14, 6, 20, 4, 8, 2, 11, 10, 14, 8, 8, 5. Find the mean, mode and median distances.

23. A group of 20 students were asked the number of pets they had. The results were 1, 0, 2, 3, 4, 2, 1, 2, 1, 1, 5, 2, 3, 1, 1, 0, 2, 1, 2, 3. Find the mean, median and mode number of pets.

24. The ages of ten teachers at a school are 34, 43, 30, 56, 37, 28, 25, 35, 40, 42. Calculate the mean and median age of the teachers.

25. The weights to the nearest kg of students in a rugby team were 48, 49, 52, 55, 64, 52, 45, 52, 60, 64, 72, 70, 68, 65, 50. Calculate the mean and median weight of the students.

26. Tatum sold muffins each lunchtime at school as part of a fundraiser for the Hockey 1st XI. Over a ten day period she sold 28, 45, 32, 55, 32, 40, 40, 29, 38 and 45 muffins. Calculate the mean, median and mode number of muffins sold over the ten day period.

27. The scores of Jake's last 15 rounds of golf have been, 92, 90, 85, 90, 99, 86, 102, 90, 94, 88, 96, 100, 99, 101 and 90. Calculate the mean, median and mode of his golf scores.

28. The test results of four students were 80, 70, 90 and x. If the mean of the test results was 80, what is the value of test score x?

29. A group of 20 students sold tickets as a fundraiser. The mean number of tickets sold per students was 85. How many tickets were sold altogether?

30. A sports shoe store is offering a special on a particular running show. In the first week of the sale it sold the following sizes of the shoe on special 6, 7, 8, 8, 8, 8, 9, 9, 9, 10, 10, 11. Of the three averages which one would be the most useful to the manager of the store. Calculate that average only and justify why.

Merit – Answer the following questions.

35. A plumber receives the following number of calls per day over a period of 33 days. See the frequency table below.

Find the mean, mode and median number of daily calls the plumber receives.

Calls per day (x)	Frequency f	x.f
1	3	3
2	3	6
3	2	6
4	5	
5	3	
6	4	
7	6	
8	3	
9	4	
TOTAL	33	

36. A marksman receives the following scores after 30 shots at a target. See the frequency table below.

Find the mean, mode and median score the marksman gets.

Score (x)	Frequency f	x.f
0	2	0
5	5	25
10	8	
15	6	
20	5	
25	4	
TOTAL	30	

37. Louise records the ages (to the nearest year) of all the children on her school bus.

Find the mean, mode and median age of the children on the bus.

Age (x)	Frequency f	x.f
11	6	
12	9	
13	7	
14	8	
15	3	
16	2	
TOTAL	35	

38. Triple A Orchards sells apples in large bags by weight. In a sample of 25 bags the number of apples were counted and the results are represented in the frequency table below.

Find the mean, mode and median number of apples in a bag.

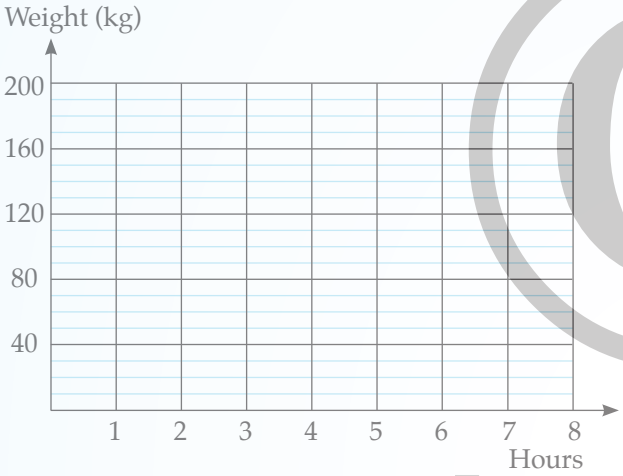
Apples per bag (x)	Frequency f	x.f
45	3	
48	3	
49	4	
50	9	
52	4	
54	2	
TOTAL	25	

57. An investigation was undertaken to see if there was a relationship between the time (hours) a person exercised for per week and their weight (kg).

Data was collected in the form (hours, weight) and is given below.

(0, 200), (4, 120), (8, 80), (0, 160), (2, 130), (2, 150), (6, 110), (7, 90), (3, 110), (1, 170), (6, 90), (1, 80), (5, 140), (5, 100), (7, 130) (8, 90), (3, 150), (1, 140), (4, 70), (4, 130), (8, 120), (2, 180).

a) Plot the data values on the set of axes below.



b) How many hours exercise per week did the heaviest person do?

c) Draw a straight line on the graph so that it 'fits' the data.

d) Describe the relationship between the number of hours exercise per week and a person's weight.

e) Do you think there are any outliers in the dataset? Give a reason for your answer.

f) What was the weight of the lightest person who exercised 4 hours per week?

g) Calculate the mean number of hours spent exercising by the people in the investigation.

58. Describe the type of relationship (negative, positive, none) you think the following investigations if undertaken would have.

a) The amount of homework a student does and the weight of their school bag.

b) The number of cars on the motorway and the time it would take to get to work.

c) The time spent talking on your cell phone and the amount of money left on the phone.

d) The age of a car and its value.

e) The income of people and the amount they spend on eating out.

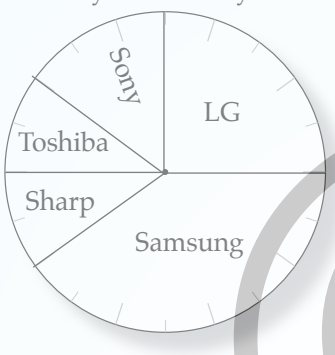
f) The number of letters in a person's name and their level of intelligence.

g) The number of years a person has been driving and the number of accidents they have had.

Achievement/Merit – Answer the following questions on pie charts.

59. An electronics store sells 5 different makes of TV. The proportion of sales by brand last month are given in the pie chart below.

Monthly TV Sales by Brand



a) The pie chart above is divided into 20 sections. What does each section represent as a percentage?

b) What was the most popular brand of TV the electronics store sold last month?

c) What was the least popular brand(s) of TV the electronics store sold last month?

d) What percentage of TVs sold by the electronics store last month were LG?

e) What percentage of TVs sold by the electronics store last month were Samsung?

f) If the electronics store sold 60 TVs last month how many of them were Samsung TVs?

g) If the electronics store sold 60 TVs last month how many of them were Sharp TVs?

h) What angle does the Samsung sector of the pie chart represent?

i) What angle does the Sharp sector of the pie chart?

60. Jake's iTunes music collection comprises 80 albums. A breakdown of the different types of albums (genres) he has are given in the table below.

Type	Pop	Rock	Country	Blues	R&B
No.	30	20	18	10	2

a) Draw a pie chart to represent Jake's iTunes music collection by first calculating the angle size for each sector.

Jake's iTunes Music

b) What percentage of Jake's music collection is pop?

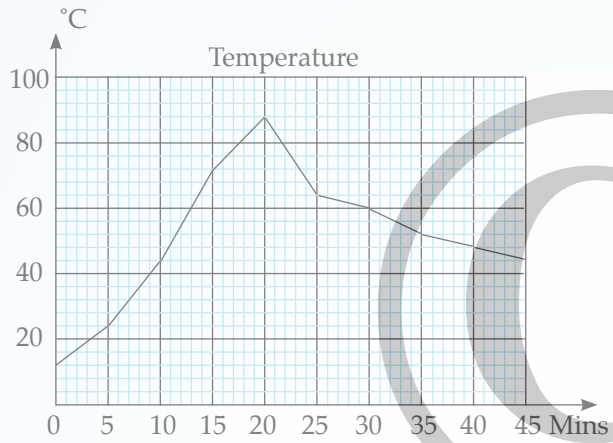
c) What two types (genre) of music albums in Jake's collection combine to give the same number as rock albums?

d) What fraction of Jake's albums are country albums?

e) What proportion of Jake's music is Blues?

Achievement/Merit – Answer the following questions on line graphs.

70. A saucepan of liquid is heated slowly over a gas flame and then allowed to cool. The temperature (°C) of the liquid is recorded every 5 minutes. A line graph of the results is drawn below.



a) What was the temperature of the liquid prior to heating?

b) What maximum temperature did the liquid reach?

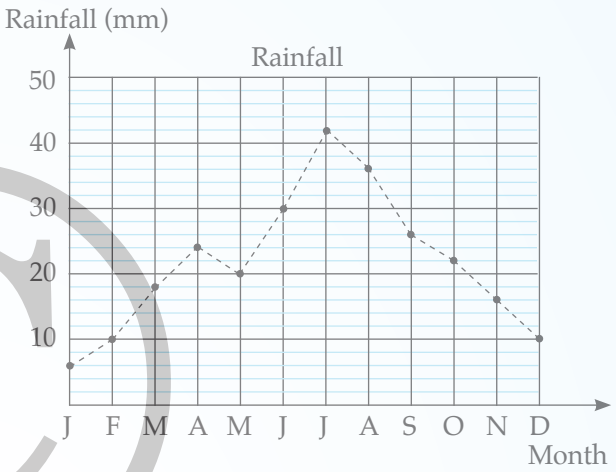
c) What was the temperature after 25 minutes?

d) What was the temperature after 45 minutes?

e) For how long was the temperature above 60 °C?

f) After how many minutes was the temperature 40 °C?

71. The line graph below gives the monthly rainfall (mm) for a town in New Zealand.



a) What month of the year was the wettest?

b) What two months had the same rainfall?

c) How many months of the year had 30 mm or more rainfall?

d) Why has a dotted line been drawn between the monthly points?

e) What was the driest month of the year?

f) How much rain fell in October?

g) What was the total rainfall throughout the year ?

Statistical Enquiry Cycle cont...

The dataset which has been provided gives the age and price of 75 Subrau Imprezas 1.5. Information on the distance (km) the cars have travelled is also included in the table, but for this investigation will not be required.

Obviously we could investigate all 75 cars in the dataset but that would take a long time. A better approach is to take a sample of the data. A sample is a smaller selection that hopefully gives us results and values similar to those if we investigated all 75 cars.

The size of a sample is important. If it is too small it may not accurately reflect the dataset as a whole. So for this investigation we are suggesting you collect a sample of 30 cars from the dataset of 75.

The data in the dataset has been numbered from 001 – 075. We can generate 30 unique numbers in the range 1 to 75 inclusive using the random generating function on the calculator.

Data

On the Casio to generate a random number between 1 and 75 inclusive we press

Ran#

7 5 x SHIFT . + 1 =

which will generate a random number each time the = button is pressed. You will need to ignore the digits after the decimal point.

On the TI we enter

RAND

prb ▶ 2 1 2nd . 7 5) enter

which will generate a random number each time you press the enter key.

As you generate your random numbers shade the left hand square corresponding to the applicable entry in the table. If a random number is repeated, ignore it. Continue until you have 30 entries in the dataset table shaded. This is you sample.

	No.	Year	Age in 2010	km (000s)	Price ($)
	043	2010	0	1	35000
	044	1996	14	172	6000
	045	2001	9	27	12000
	046	1999	11	88	9000
	047	2002	8	115	9000
	048	2001	9	92	7500
	049	2001	9	94	9000
	050	2000	10	59	9000
	051	2001	9	120	9000
	052	2002	8	50	14000
	053	2003	7	68	11000
	054	2004	6	104	11000
	055	2001	9	70	10000
	056	2005	5	95	9000
	057	1994	16	200	3000
	058	2006	4	89	13000
	059	2002	8	105	10000
	060	2003	7	75	11000
	061	2004	6	42	13000
	062	2002	8	71	10000
	063	2002	8	103	8000
	064	2007	3	54	18000
	065	2004	6	84	12000
	066	2000	10	86	11000
	067	2002	8	95	10000
	068	2001	9	41	10500
	069	2001	9	104	9000
	070	2001	9	87	10000
	071	1999	11	54	9000
	072	2000	10	92	9000
	073	1996	14	110	6500
	074	1993	17	85	5000
	075	1998	12	100	4000

On the Casio calculator we multiplied the Ran# function by 75 and added 1. The reason we added one is because 75 x Ran# on its own would have produced a number from 0 to 74 inclusive. By adding one we get a number from 1 to 75 inclusive.

Achievement – Answer the following questions.

6. Four different coloured marbles are placed in a bag. A marble is drawn out and its colour noted. It is then placed back in the bag and another marble drawn out. The process is repeated 100 times. The results of the experiment are given in the table below.

	No. of times
Red	32
Green	18
Blue	21
Yellow	29
Total	100

a) List the sample space (possible outcomes) of the experiment.

b) Calculate the relative frequency of a red marble being drawn out.

c) Calculate the relative frequency of a yellow marble being drawn out.

d) Calculate the relative frequency of a blue or green marble being drawn out.

e) If the experiment had been conducted 200 times what do you think the relative frequency of a yellow marble being drawn out would be?

7. A coin and a die are thrown at the same time repeatedly. The results are given in the table below.

	H	T	Total
1	6	10	16
2	10	7	17
3	8	9	17
4	13	10	23
5	6	15	21
6	2	4	6
Total	45	55	100

a) List the sample space (possible outcomes) of the experiment. The first one is done for you.

H, 1

b) How many times was the coin and die thrown during the experiment?

c) How many times did the coin land heads up during the experiment?

d) How many times did a six land face up on the die during the experiment?

e) Calculate the relative frequency of getting a four on the die and a tail on the coin.

f) Calculate the relative frequency of getting a three on the die and a head on the coin.

g) If the experiment had been conducted 300 times, what do you think the relative frequency of a five and tail would be?

Example

a) A coin is thrown first followed by a die.

Draw a tree diagram to represent this. Put all the probabilities on your tree.

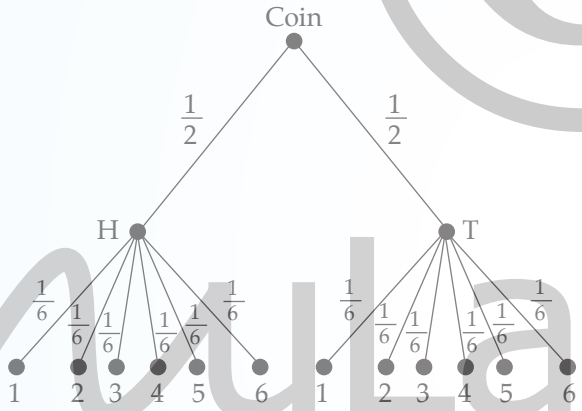
b) Using your tree diagram from part a) list all the possible outcomes (sample space).

c) Using your tree diagram find the probability of getting a head followed by the number 5.

d) Using your tree diagram find the probability of getting a tail followed by an even number on the die.

Solution

a) Tree diagram showing probabilities and outcomes.



Outcomes

H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

b) Possible outcomes:

H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

c) To find the probability of a head followed by the number 5 we put the number of times H5 occurs (1) over the total number of possible outcomes (12) so

$$P(\text{head followed by 5}) = \frac{1}{12}$$

d) To find the probability of a tail followed by an even number we put the number of times a tail followed by an even number occurs (T2, T4, T6) = 3) over the total number of possible outcomes (12) so

$$P(\text{tail followed by an even number}) = \frac{3}{12}$$

Extra Examples

By reading from the top of the probability tree and down each pair of branches in turn we can list all the possible outcomes, e.g. H1, H2 etc.

7.5 Place Value Strategy (Subtraction)

Place Value

The place value strategy involves choosing the largest number and then successively subtracting the thousands, hundreds, tens and ones of the other number from it. Sometimes this method is also called Separating or Decomposing.

When is the ideal situation to use the place value strategy for subtraction?

Use the place value strategy when the digits of the smaller number are less than or equal to the corresponding digits in the larger number. Note: In some instances place value is still worthwhile using if this is not the case.

The problem 357 – 245 is ideal for using the place value strategy, but not so for 324 – 187 because (7 > 4 and 8 > 2).

Once we have identified that the strategy we plan to use is place value choose the largest of the numbers as your starting number and successively subtract the thousands, hundreds, tens and then the ones of the other number, from it.

Consider the subtraction problem 549 – 214.

Step 1 – choose the largest number as your starting number e.g. 549.

Step 2 – expand the second number i.e. 216 = 200 + 10 + 4.

Step 3 – subtract 200, then 10, then 4 from your the starting number.

Your processing or thinking would be as follows:

Starting number → 549 – 200 = 349 ← Subtracting 100s

Total so far → 349 – 10 = 339 ← Subtracting 10s

Total so far → 339 – 4 = 335 ← Subtracting 1s

Answer

Many of the questions you will be asked in this section can be solved using multiple strategies not just place value.

Example

Use the place value strategy to answer the following questions.

a) 68 – 35 =

b) 453 – 125 =

c) 4876 – 2634 =

Question	Strategy	Thinking
a) 68 – 35 =	*Place value*	68 – 30 = 38 38 – 5 = 33
b) 453 – 125 =	*Place value*	453 – 100 = 353 353 – 20 = 333 333 – 5 = 328
c) 4876 – 2634 =	*Place value*	4876 – 2000 = 2876 2876 – 600 = 2276 2276 – 30 = 2246 2246 – 4 = 2242

According to a USA survey 60% of customers who were going to purchase an iPad, were planning to purchase the 16 GB, Wi-Fi + 3G model costing $629 US.

(d) If 300 000 iPads were sold in the first day of sales, how many would you have expected to be the 16GB, Wi-Fi + 3G model?

(e) One megabyte is equivalent to approximately 1 049 000 bytes and one gigabyte is approximately 1000 megabytes. If an iPad data plan gives a user 2 GB of data per month, how many bytes is that? Give your answer in standard form.

(f) $\frac{13}{20}$ of the customers who purchase an iPad take out the extended online technical support option and $\frac{2}{7}$ of these end up having to use it. If 420 customers purchase an iPad how many end up using the extended online technical support?

(g) Jake buys an iPad for $979 on hire purchase. He pays a deposit of $150 and 12 monthly payments of $85. How much more does he pay for the iPad by using hire purchase?

Page 41

366. $\square = 42$
367. $\square = 96$
368. $\square = 64$
369. $\square = 72$
370. $4\frac{1}{2}$
371. a) $\frac{1}{30}$
b) $1\frac{1}{2}$ km
372. 108
373. 21
374. 120
375. $\frac{6}{7}$
376. $2\frac{2}{5}$
377. $16\frac{2}{3}$
378. 184 ha
379. $600

Page 43

380. 0.2
381. 0.15
382. 0.225
383. 0.875
384. 0.95
385. 0.52
386. 0.265
387. 0.93
388. 2.04
389. 4.06
390. 3.65
391. 4.85
392. 0.003
393. 7.4

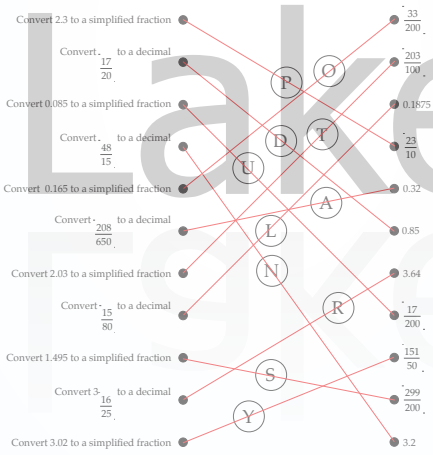
Page 44

394. $\frac{4}{5}$
395. $\frac{3}{4}$
396. $\frac{1}{20}$
397. $\frac{2}{5}$
398. $\frac{13}{20}$

Page 44 cont...

399. $\frac{9}{125}$
400. $\frac{37}{200}$
401. $\frac{2}{25}$
402. $\frac{3}{25}$
403. $\frac{9}{5}$
404. $\frac{51}{20}$
405. $\frac{89}{20}$
406. 0.125
407. $\frac{7}{20}$
408. 0.8125
409. $\frac{13}{25}$
410. 0.575
411. 0.075

Page 45



Why did the parrot wear a raincoat?

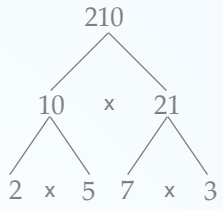
SO HE COULD BE POLYUNSATURATED

Page 47

412. 5, 19
413. 31, 37, 41, 43
414. 2, 5, 13, 29, 47
415. 53, 59
416. 11, 23, 61, 67
417. 71, 73, 79
418. 3, 17, 83, 89
419. 97, 101
420. 103
421. 131, 137, 139

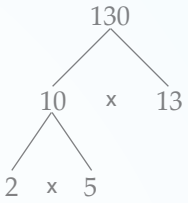
Page 47 cont...

422.



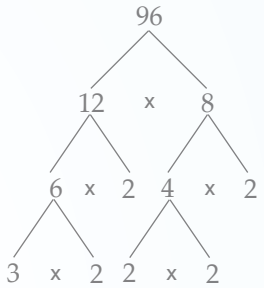
2 x 3 x 5 x 7

423.



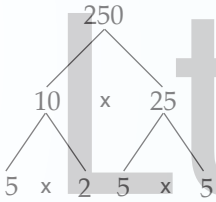
2 x 5 x 13

424.



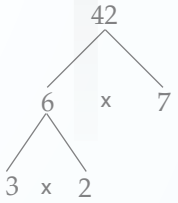
2 x 2 x 2 x 2 x 2 x 3

425.



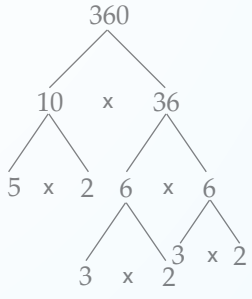
2 x 5 x 5 x 5

426.



2 x 3 x 7

427.



2 x 2 x 2 x 3 x 3 x 5

Topic 4

Shape

Page 162

1. ∠RTQ. Obtuse – greater than 90° and less than 180°.

2. ∠DGK. Right angle – equal to 90°.

3. ∠JWS. Acute – between 0° and 90°.

4. ∠BAC. Straight angle – equal to 180°.

5. ∠PAF. Reflex – greater than 180° and less than 360°.

6. ∠TMZ. Obtuse – greater than 90° and less than 180°.

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7. ∠ABC = 65°

8. ∠DEF = 155°

9. ∠GHI = 64°

10. ∠JKL = 177°

11. ∠MNO = 92°

12. ∠PQR = 128°

13 ∠ABC = 55°

14. ∠DEF = 15°

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15. ∠GHI = 165°

16. ∠JKL = 140°

17. ∠MNO = 74°

18. ∠PQR = 235°

19. ∠STU = 92°
∠UTV = 50°

20. ∠WXY = 92°
∠YXZ = 72°

21. ∠ABC = 22°
∠ABD = 100°

22. ∠HEF = 121°
∠FEG = 31°

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23. a = 142°

24. b = 114°

25. c = 42°

26. d = 38°

27. e = 12°

28. f = 58°

29. g = 18°

30. h = 15°

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31. i = 134°

32. j = 61°

33. k = 108°

34. m = 324°

Page 168 cont...

35. n = 66°

36. p = 72°

37. q = 31°

38. r = 97°

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39. s = 127°
t = 53°

40. u = 94°
v = 86°

41. w = 157°
x = 23°
y = 23°

42. z = 75°
a = 105°
b = 105°

43. c = 74°

44. d = 19°
e = 85°

45. f = 37°
g = 143°

46. h = 28°
i = 152°
j = 152°

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47. m = 90°
Adj. ∠s on a str. line = 180°.

48. n = 52°
Adj. ∠s on a str. line = 180°.
o = 46°
Adj. ∠s on a str. line = 180°.

49. p = 120°
∠s at a point = 360°.

50. q = 18°
Adj. ∠s on a str. line = 180°.

51. r = 29°
Vert. opposite ∠s are equal.
s = 151°
Adj. ∠s on a str. line = 180°.

52. t = 27°
Adj. ∠s on a str. line = 180°.
u = 27°
Vert. opposite ∠s are equal.

53. v = 36°
Vert. opposite ∠s are equal.
w = 43°
Vert. opposite ∠s are equal.
x = 101°
Adj. ∠s on a str. line = 180°.

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54. a = 87°

55. b = 28°

56. c = 29°
d = 151°

57. e = 103°
f = 147°
g = 77°

58. h = 58°
i = 152°
j = 86°
k = 122°

59. m = 63°
n = 63°
p = 117°

60. q = 37°
r = 59°
s = 143°

61. t = 39°

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62. a = 141°

63. b = 117°

Page 174

64. c = 65°

65. d = 29°

66. c = 83°
d = 97°

67. e = 66°
f = 126°

68. g = 98°
h = 134°

69. i = 89°
j = 146°

70. k = 93°
l = 93°

71. m = 56°
n = 37°

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72. a = 137°
b = 43°

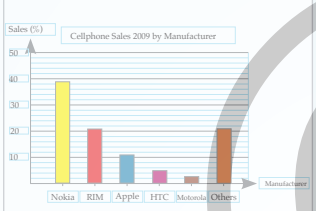
73. c = 64°
d = 116°

74. e = 93°
f = 93°

75. g = 39°
h = 141°

76. m = 111°
n = 111°
p = 69°

Assessment Schedule – Statistics and Probability – Pages 338 – 340

Question No.	Achievement	Merit	Excellence	Justification
	Carry out statistical graphing and calculations for data and determine probabilities.	**Carry out statistical processes and determine probabilities**	**Solve theoretical probability problems and comment on significant features shown by comparative graphs.**	
1 (a)	Bar graph with all boxes filled in correctly.  			**A1** **Allow one minor error.**
1 (b)	1 801 800			**A1**
1 (c) (i)	87 texts			**A1**
1 (c) (ii)		1891 ÷ 23 = 82.2 texts per day		**M** **Accept 82 or 83. Working required.**
2 (a)	0.75			**A2**
2 (b)	Prob. = 0.25 x 0.20 = 0.05	Num. = 320 x 0.05 = 16 students		**A2/M**
2 (c)	Prob. = 0.75 x 0.90 = 0.675			**A2**
3 (a)			No, spending more in 2009 than in 2003. In 2003 people were spending $900 per year, whereas in 2009 they were spending $1100.	**E** **Require correct conclusion plus figures to back up conclusion.**
3 (b)	Less familiar with technology.	Less disposable income. Less influenced by latest trends. Over 65's usually have a landline.		**A2/M** **One valid reason for A2, two valid reasons for M.**
Sufficiency	**2 or 3 of A1 and 2 or 3 of A2**	**Achievement plus 1M or 2M**	**Merit plus E**	